DOUBLE BALANCED BILATERAL
RING MODULATOR

R. D. CLUBB

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PREFACE

This paper is primarily concerned with the double balanced ring modulator using germanium crystal diodes as the rectifying elements. However, some of the basic circuit configurations will be given in a general discussion of rectifier modulators. The remainder of the paper will be devoted to the transmission properties of the ring modulator. Although the history of these modulators is long, their design has been largely empirical with little serious attempt to analyze the finer points of the circuit operation with a view of improving them. It is the object of this paper to investigate some of these details.

The work included here was done under the supervision of Professor G. R. Giet, whose criticism and editorial comment is gratefully acknowledged. The writer also wishes to express his appreciation to Mr. J. F. Honey, Mr. D. K. Weaver and Dr. F. Clelland for their assistance in work on the double balanced ring modulator undertaken by the writer at the Stanford Research Institute.

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TABLE OF SYMBOLS AND ABBREVIATIONS

a.c. - alternating current

α - rectifier resistance law constant

c - carrier input frequency in radians per second

c - subscript referring to carrier source

d.c. - direct current

Δ - an increment $\ll 1$

db - decibels

e - instantaneous value of voltage

f - frequency in cycles per second

f(t) - function of time

γ - propogation constant

i - instantaneous value of current

I - alternating current

∞ - infinity

k - rectifier resistance law constant

L - insertion loss ratio

log - logarithim to the base 10

m - rectifier resistance law constant

m - ratio of circuit impedance to optimum value

n - rectifier resistance law constant

n^2 - ratio of backward to forward resistance

n_e - any even number

n_o - any odd number

η - efficiency

o - subscript referring to initial value before change

p - ratio of backward resistance variations to forward resistance variations

π - 3.1416 radians

R - d.c. resistance

R_f - d.c. resistance of crystal diode in the forward direction of current flow

R_b - d.c. resistance of crystal diode in the backward direction of current flow

r - a.c. resistance

r_f - a.c. resistance of crystal diode when biased in forward direction

r_b - a.c. resistance of crystal diode when biased in backward direction

S - peak value of sinusoidal input signal

s - signal input frequency in radians per second

s - subscript referring to input signal

t - time in seconds

v - instantaneous value of voltage across a rectifier

w - angular frequency in radians per second

φ - modulating function

$\varphi(t)$ - modulating function

Z - complex impedance in ohms

SUMMARY

A brief general discussion of rectifier modulators is given. This is followed by a description of the principle of operation of the ring modulator under idealized "perfect switch" conditions. Though the "perfect switch" assumption is useful mainly in qualitative descriptions of circuit operation, useful information is also obtained regarding circuit stability. The concept of a modulating function is introduced and used as a basis of analysis under the assumption of linear circuit elements. Resistance functions to represent the actual resistance of the crystal rectifier are discussed and used with the modulating function to give an approximate non-linear analysis of the circuit.

I. RECTIFIER MODULATORS

These crystal diode modulators probably differ most from vacuum tube modulators in that the simplicity of the rectifier elements allows a greater variety of circuit arrangements to be used. Also, unlike vacuum tube modulators, the rectifier modulators transmit signals equally well in either direction. This is a simplification which allows a modulator to be used as a demodulator.

The circuit arrangements used in crystal diode modulators are generally concerned with balancing action to suppress some unwanted frequency in the signal output. In single sideband applications it is desired to suppress the carrier and obtain the desired sideband from the output by means of a suitable filter. If the circuit is to be used bilaterally, i.e., as a modulator on transmit and a demodulator on receive, it may be desireable to suppress the carrier at both the input signal terminals and the output signal terminals.

Figure 1 illustrates four possible circuit arrangements. In Figure 1(a), 1(b), and 1(c) the circuit is arranged to suppress the carrier in both the signal input and signal output circuits. In Figure 1(d) the carrier is suppressed in only one signal branch. Ideally, under conditions of perfect balance, the carrier would be completely balanced out, but in practice some carrier voltage appears in the output.

In any of the circuits shown, modulation results from

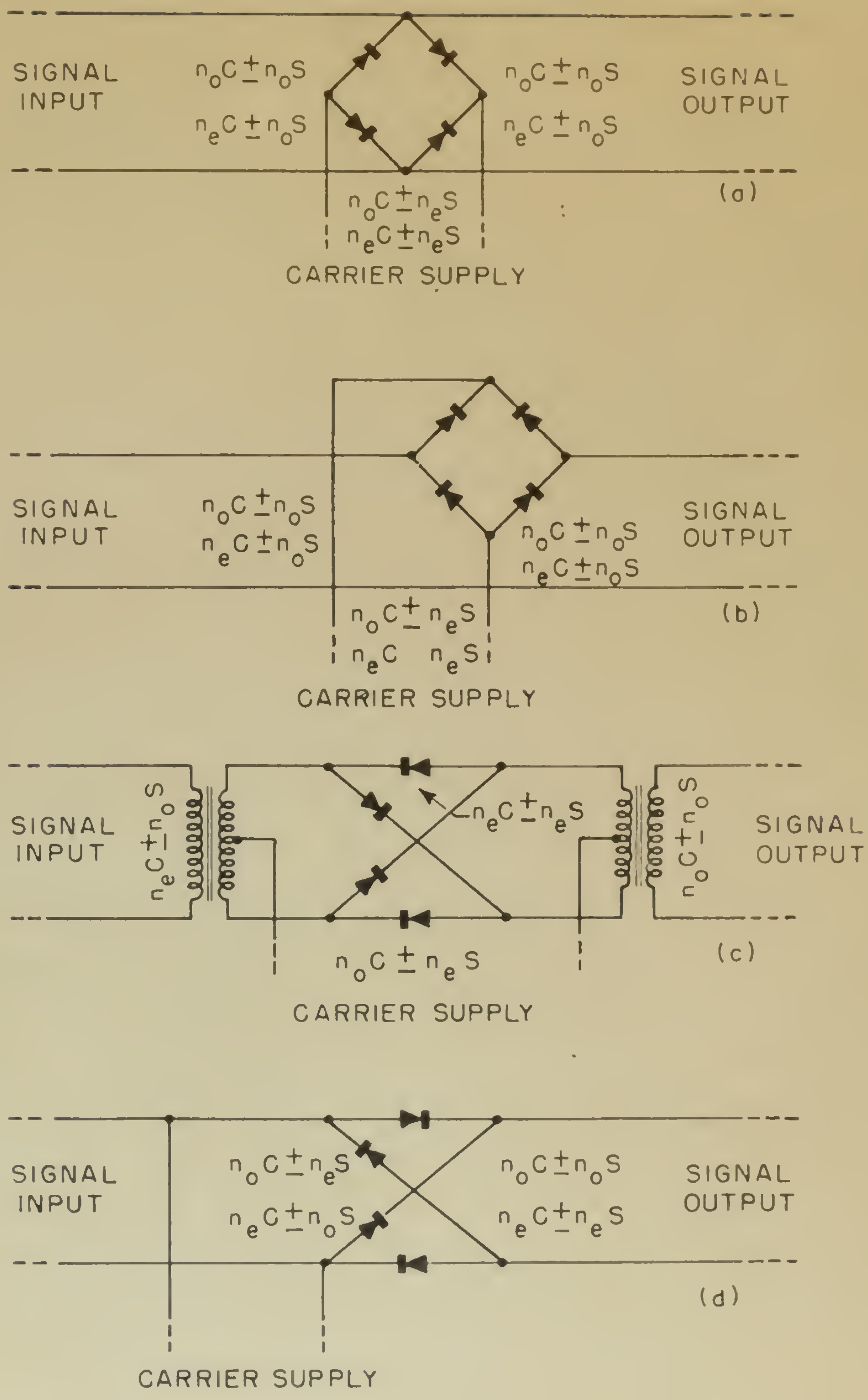

Figure 1

TYPES OF CRYSTAL DIODE MODULATORS

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either the reduction or reversal of current flow between the input and output signal circuits at periodic intervals as the carrier varies the crystal diode resistance back and forth between high and low values.

In the usual applications selective filters must be used in each signal branch to restrict transmission to the desired frequency band. In Figure 1(a) where the input and output circuits are periodically short circuited by the carrier actuated crystal diodes, transmission of the modulated signal into the input circuit or the unmodulated signal into the output circuit is prevented by filters, each having a high impedance at the frequency of the other signal. In Figure 1(b) the filter should have a low impedance at the other signal frequency since the connections between the input and output circuits are periodically open-circuited by the switching function of the carrier. This type of arrangement is sometimes referred to as the series modulator while that of Figure 1(a) is referred to as the shunt modulator. In Figure 1(c) and 1(d) the crystal diodes are made to become alternately low and high resistance in pairs as the polarity of the carrier is either in the same direction as the arrows or in the opposite direction. The arrangement of Figure 1(c) is variously referred to as a ring, double balanced, or reversing switch modulator.

In these rectifier modulators all modulation product frequencies can be grouped into four classes:

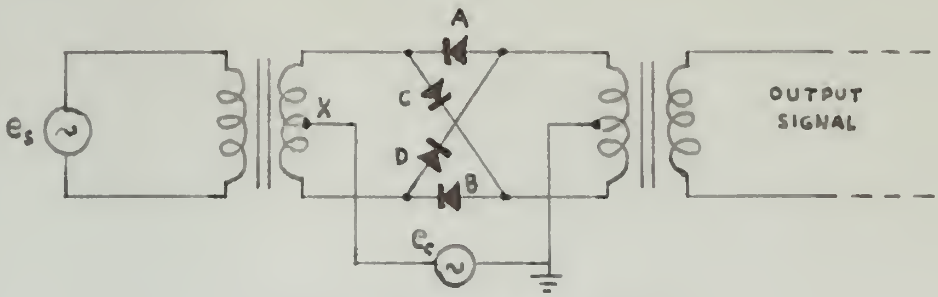
$$n_o c \pm n_o s \qquad n_o c \pm n_e s \qquad n_e c \pm n_o s \qquad n_e c \pm n_e s$$

in which c and s are carrier and input signal frequencies and n_o is any odd number while n_e is any even number. The branches in which these modulation products appear are shown in the circuits illustrated. In the double balanced circuit of Figure 1(c) these types of products are completely separated in different parts of the circuit while in all the other circuits the classes of products appear together in combinations of two types.

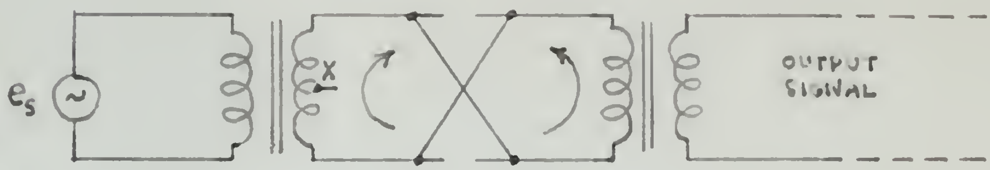
II. DOUBLE BALANCED RING MODULATOR

An understanding of the operation of the double balanced ring modulator may most easily be obtained by assuming that the rectifier elements are perfect rectifiers with zero forward resistance and infinite backward resistance and that the transformers are perfectly balanced.

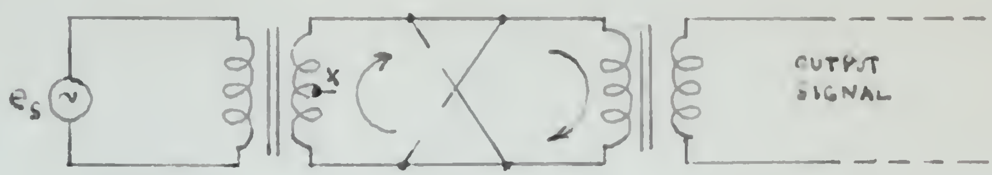
On the basis of this assumption equivalent circuits may be drawn as in Figure 2. When the polarity of the



Balanced Ring Modulator



Equivalent Circuit During Positive Half Cycle



Equivalent Circuit During Negative Half Cycle

Figure 2

carrier is such that point x is positive, diodes A and B will be open circuits while diodes C and D will Conduct, becoming short circuits. During the other half cycle when point x is negative, diodes A and B will be short circuits while diodes C and D are open. Thus the carrier acts as a double-pole double-throw switch which reverses the signal current at the carrier frequency.

An expression for this switching function is simply the Fourier expansion for a square wave

$$e_c = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos(2n+1)ct \qquad (1)$$

where the amplitude of the square wave is assumed to be unity. Suppose the lattice is supplied from a signal source,

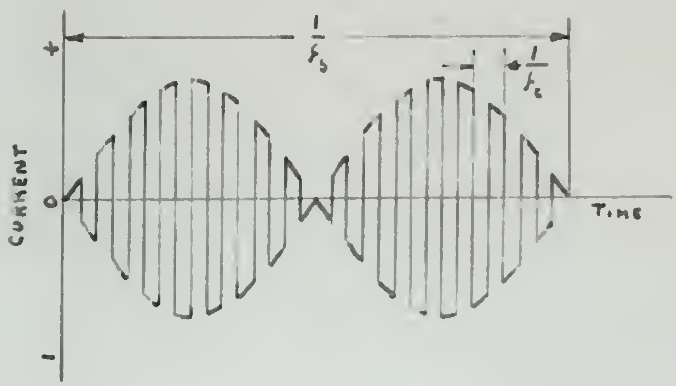


Figure 3. Output Current of Balanced Ring Modulator

$e_s = S\cos st$, of internal impedance R_1 and is terminated by the impedance R_2. Then the input signal will be multiplied by the switching, or modulating, function to produce the output current of Figure 3. The current will be given by

$$I = \frac{4S\cos st}{\pi(R_1+R_2)} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cos(2n+1)ct \qquad (2)$$

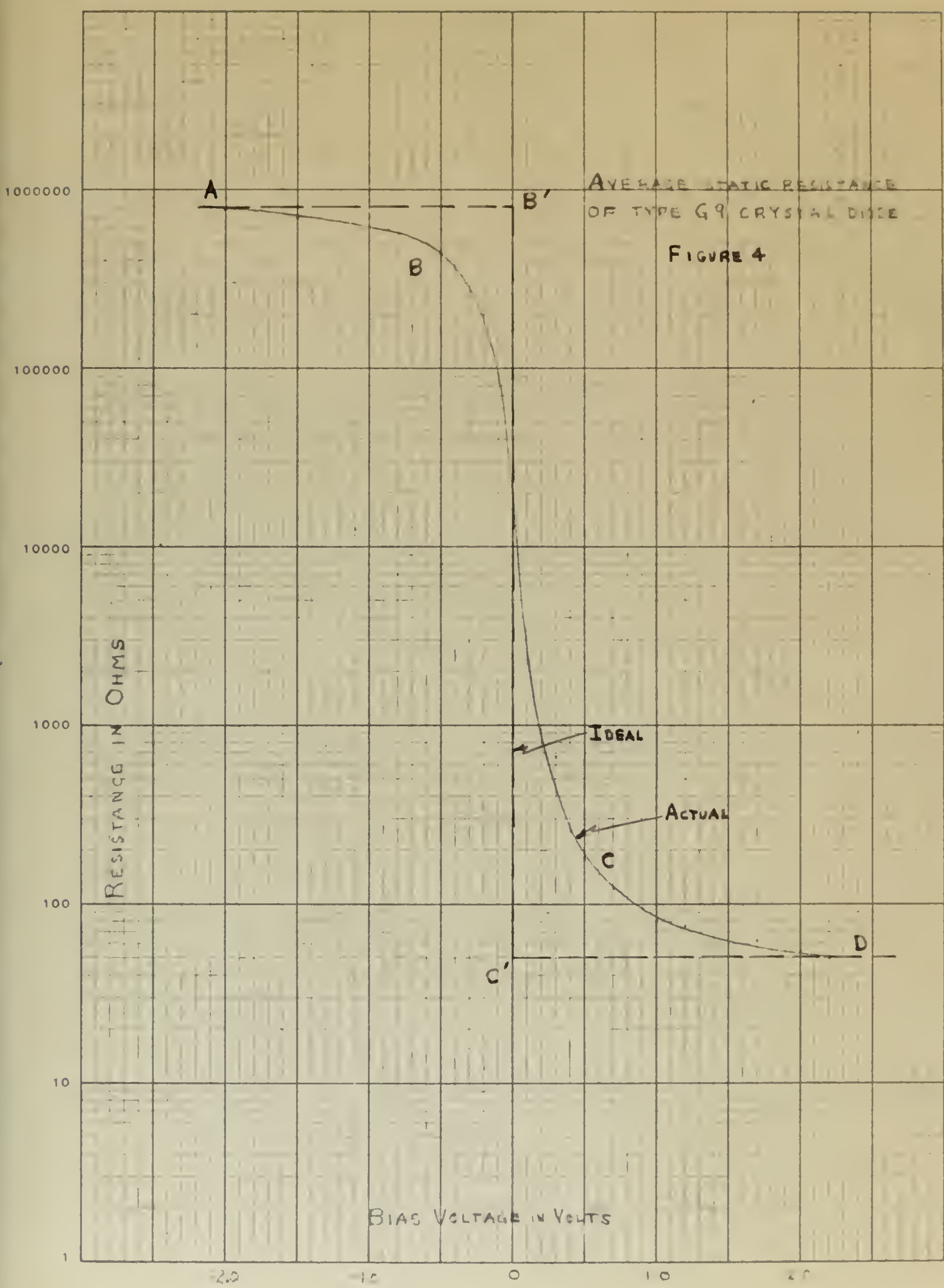
$$I = \frac{2S}{\pi(R_1+R_2)} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \left\{ \cos\left[(2n+1)c+s\right]t + \cos\left[(2n+1)c-s\right]t \right\} \qquad (3)$$

From this equation it can be seen that the output current contains only sideband frequencies. The carrier and the unmodulated input signal are not present.

In the foregoing it was assumed that the crystal diodes acted as perfect rectifiers and were perfectly balanced in the bridge configuration. In practice this ideal condition can only be approximated. The diodes do not actually present zero resistance to the transmission of current in one direction and infinite resistance to the flow of current in the opposite direction. Nor, as may be seen from Figure 4, is the transition from a high resistance to a low resistance as sharp as might be desired. Exact balance of the four rectifier elements and the transformers is also a condition which can only be approached in practice.

As a result of the above practical facts, the modulator output always contains numerous components in addition to those indicated by equation (3) including the carrier frequency itself. Most troublesome of these unwanted frequencies are harmonics of the signal frequencies and cross modulation products which fall within the range of the desired sideband frequencies and thus cause distortion. The other unwanted frequencies can be eliminated by means of suitable filters.

MODEL DATE



AVERAGE STATIC RESISTANCE
OF TYPE G9 CRYSTAL DIODE
FIGURE 4
A
B
B′
C
C′
D
IDEAL
ACTUAL
RESISTANCE IN OHMS
BIAS VOLTAGE IN VOLTS
1000000
100000
10000
1000
100
10
1
2.0
0
1.0

To obtain this switching action using the rectifier elements, it may be seen from the above discussion that the carrier voltage must be large enough to insure that the diodes are switched rapidly from a definitely low resistance to a definitely high resistance. Also, the input signal must be small with respect to the carrier so that it will have negligible effect on the switching action and so that the signal swing will be confined to a linear portion of the diode current-voltage characteristic. In this way, the resistance presented by the rectifier elements will be under control of the carrier alone.

In summary, the carrier may be thought of crudely as acting as a d.c. bias voltage determining the operating point on the diode characteristic and at the same time reversing the signal current at the carrier frequency.

III. THE PERFECT SWITCH MODULATOR

The concept of a modulating function* by which an input signal is multiplied to produce the output signal is useful as a basis for analysis of the modulator performance. While the modulating function for the perfect switch case is primarily of value in qualitative investigations of circuit behavior, further results can be obtained by a consideration of this modulating function in a linear analysis of the circuit.

Assume now that the crystal diodes have a constant low resistance for one direction of current flow and a constant high resistance for current flow of the opposite direction. That is, assume that the resistance characteristic for the crystal rectifier is the ideal represented by curve AB'C'D of Figure 4.

The circuit then reduces to an approximate equivalent linear system which is approached in practice by using a large carrier amplitude and making the signal sufficiently small compared to the carrier that it can be varied in magnitude without noticeable effect on the signal impedance or on the linearity between input and output signal amplitudes.

The approximate equivalent linear circuit of Figure 5 may then be drawn to represent the ring modulator, where R_f is the constant low forward resistance of the crystal rectifier, R_b is the constant high crystal back resistance and Z_1 and Z_2 are the impedances of the terminating circuits.

*D. G. Tucker (3)

No attempt will be made in this paper to consider the shunt capacity of the rectifier elements, for the mathematical complexity caused by such a consideration makes the problem practically impossible to solve analytically and the design of a modulator in which the shunt capacity of the rectifier elements cannot be neglected must be empirical. Caruthers* gives some discussion of the subject. However, there seems to be little need to consider the shunt capacity

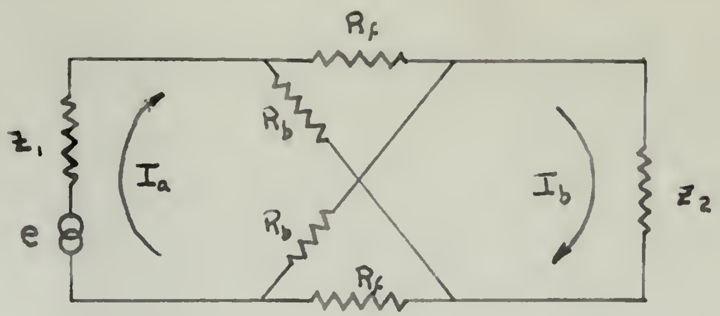


Figure 5. Equivalent Linear Ring Modulator Circuit.

since crystal diodes are available with shunt capacity of the order of $1\mu\mu f$ or less. These rectifiers may be considered essentially resistive up to frequencies of the order of one megacycle per second; they also remain efficient rectifiers up to several hundred megacycles per second.

Consideration of the mesh equations for Figure 5 gives

$$I_b = \frac{2e(R_b - R_f)}{(2Z_1 + R_b + R_f)(2Z_2 + R_b + R_f) - (R_f - R_b)^2} \tag{4}$$

*R. S. Caruthers (2)

Now I_b with the lattice removed, i.e., $R_b = \infty$ and $R_f = 0$, is given by

$$I_b = \frac{e}{Z_1 + Z_2} \tag{5}$$

so that the insertion loss ratio is

$$L = \frac{2Z_1 Z_2 + 2R_b R_f}{(R_b - R_f)\,(Z_1 + Z_2)} + \frac{R_b + R_f}{R_b - R_f} \tag{6}$$

The usual case is for $Z_1 = Z_2 = Z$.

Then

$$L = \frac{(Z + R_b)\,(Z + R_f)}{Z(R_b - R_f)} \tag{7}$$

or the modulating function is

$$\varphi = \frac{1}{L} = \frac{Z(R_b - R_f)}{(Z + R_b)\,(Z + R_f)} \tag{8}$$

The value of Z for which this is a maximum is obtained by differentiating with respect to Z and equating to zero. This gives

$$(Z + R_b)\,(Z + R_f)\,(R_b - R_f) = Z(R_b - R_f)\,(2Z + R_b + R_f) \tag{9}$$

which reduces to

$$Z = \sqrt{R_b R_f} \tag{10}$$

Thus the transfer function will be a maximum for external

circuit terminations equal to the characteristic impedance of the lattice.

Substituting this optimum value of Z into (8)

$$\varphi_{max} = \frac{\sqrt{R_b R_f}\ (R_b - R_f)}{2R_b R_f + \sqrt{R_b R_f}\ (R_b + R_f)} \tag{11}$$

$$\varphi_{max} = \frac{\sqrt{\frac{R_b}{R_f}} - 1}{\sqrt{\frac{R_b}{R_f}} - 1} \tag{12}$$

letting

$$n^2 = \frac{R_b}{R_f} \tag{13}$$

$$\varphi_{max} = \frac{n-1}{n+1} \tag{14}$$

and

$$I_b = \frac{e}{2Z} \cdot \frac{n-1}{n+1} \cdot f(t) \tag{15}$$

where the factor f(t) has been added in equation (15) to account for the reversal of current due to the switching action of the carrier.

$$f(t) = \begin{cases} +\ 1 \text{ for positive half cycle of the carrier} \\ -\ 1 \text{ for negative half cycle of the carrier} \end{cases}$$

Letting

$$f(t) = \frac{4}{\pi}\left[\cos ct - \frac{1}{3}\cos 3ct + \frac{1}{5}\cos 5ct \text{ -------}\right] \tag{16}$$

$$I_b = \frac{e\varphi}{2Z}\frac{4}{\pi}\left[\cos ct - \frac{1}{3}\cos 3ct + \frac{1}{5}\cos 5ct \text{ -------}\right] \tag{17}$$

The magnitude of the desired single sideband output current is then

$$I_{1+} = I_{1-} = \frac{e}{2Z}\frac{2\varphi}{\pi} \tag{18}$$

and the power efficiency

$$\eta_{max} = \frac{4}{\pi^2}\varphi^2 \tag{19}$$

As may be seen from equation (17) currents also flow at both sidebands of all odd harmonics of the carrier frequency. No currents flow at the side band frequencies of the even harmonics of the carrier.

Calculations of the variation of efficiency with n for this modulating function are plotted in Figure 6. For most crystals $n \geq 1000$ so that the ratio of back to front resistance is not critical as regards efficiency.

Consider now the general case where $Z = m\sqrt{R_b R_f}$ Then from (8)

$$\varphi = \frac{m\sqrt{R_b R_f}\,(R_b - R_f)}{(m\sqrt{R_b R_f} + R_b)\,(m\sqrt{R_b R_f} + R_f)} \tag{20}$$

$$\varphi = \frac{n^2 - 1}{(1 + mn)\,(1 + n/m)} \tag{21}$$

The decrease of efficiency when the circuit impedance is changed from the optimum value will be given by

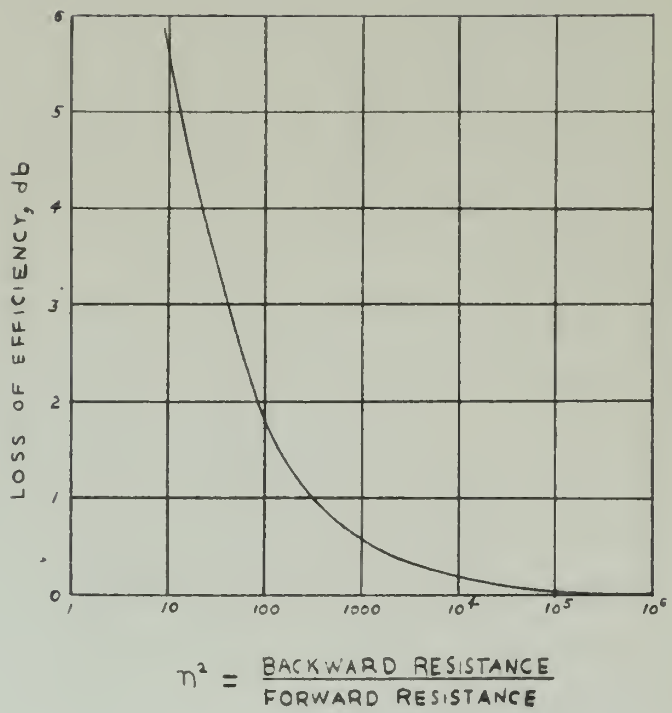


Figure 6. Effect of n on efficiency of a ring modulator (For case of optimum circuit impedance).

$$\text{Decrease in efficiency} = 20\log\frac{\text{optimum efficiency}}{\text{non-optimum efficiency}}$$

$$= 20\log\frac{(1+mn)\,(1+n/m)}{(n+1)^2} \qquad (22)$$

This is computed and plotted in Figure 7 for various values of n. It can be seen that, in general, the value of the terminating circuit impedance is not critical.

Differentiating the ratio term in (22) shows that the rate of change of efficiency with m is zero when $m = 1$. Similarly the rate of change of efficiency with n is zero when $n = \infty$. Also the limit of the ratio as n approaches infinity is unity for any finite value of m showing that the larger the ratio of back to front resistance of the rectifier, the less will be the effect of variations of terminating circuit impedance on the modulator efficiency. The obvious conclusion to be drawn from this discussion is that the stability of the modulator efficiency is highest when the modulator is designed for maximum efficiency.

In germanium crystal rectifiers the variation of backward resistance with temperature will be much greater proportionately than the variation of forward resistance. Also the variation of backward resistance from one rectifier to another within a sample may be much greater than the variation of forward resistance. Therefore this is the condition for which stability conditions are desired. The work which follows is patterned after the method of Tucker*.

*D. G. Tucker (3)

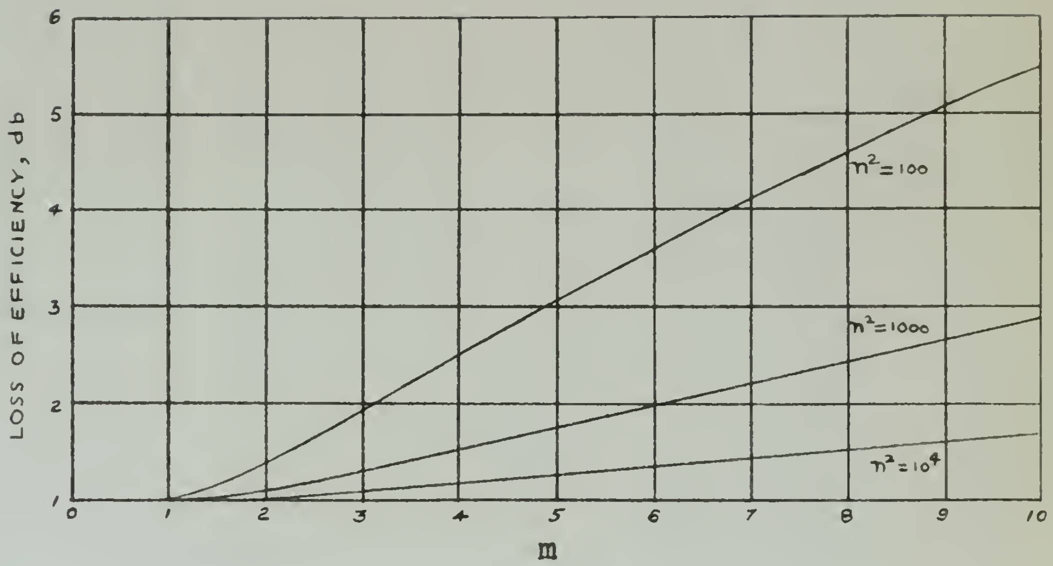


Figure 7. Effect of circuit impedance on efficiency of ring modulator.

Let the forward resistance change by a small proportion Δ to $R_f(1+\Delta)$, and the backward resistance change by the proportion $p\Delta$ to $R_b(1+p\Delta)$. Assume throughout that $\Delta \ll 1$. Let the subscript o designate values before the change and the subscript Δ designate values after the change.

For the general case before the incremental change

$$Z = m\sqrt{R_b R_f} = m_o R_o \tag{23}$$

The terminating circuit impedance remains fixed during the change but R changes to

$$R_\Delta = \sqrt{R_b R_f}\,\sqrt{(1+\Delta)\,(1+p\Delta)}$$

$$R_\Delta \approx R_o\left(1+\frac{p+1}{2}\Delta\right) \tag{24}$$

Therefore m must change to

$$m_\Delta = \frac{m_o}{\left(1+\frac{p+1}{2}\Delta\right)} \tag{25}$$

to maintain the terminating circuit impedance as before.

$$n_\Delta^2 = \frac{R_b(1+p\Delta)}{R_f(1+\Delta)} = n_o^2 \cdot \frac{1+p\Delta}{1+\Delta} \tag{26}$$

$$n_\Delta \approx n_o\left[1+\frac{p-1}{2}\Delta\right] \tag{27}$$

Using the form of (21) for the modulating function

$$Q_\Delta = \frac{n_\Delta^2 - 1}{(1+m_\Delta n_\Delta)\,(1+n_\Delta/m_\Delta)} \tag{28}$$

Substituting these new values in terms of the old, the modulating function becomes

$$\varphi_\Delta = \frac{n_o^2\left[1 + (p - 1)\right] - 1}{1 + m_o n_o(1-\Delta) + \frac{n_o}{m_o}(1 + p\Delta) + n_o^2\left[1 + (p-1)\Delta\right]} \tag{29}$$

Taking $\frac{d\varphi_\Delta}{d\Delta}$ and equating to zero gives after considerable manipulation and the solution of a quadratic equation

$$m_o = \frac{-n_o(p - 1) \pm \sqrt{p}(n_o^2 - 1)}{pn_o^2 - 1} \tag{30}$$

In the last step before solution of the quadratic equation the approximation is made that terms involving Δ as well as those involving Δ^2 are negligible.

It can be seen that the positive sign is the correct one since p and m_o are positive numbers and m_o must be real and positive. Therefore the modulator is most stable with respect to small rectifier resistance variations when

$$m_o = \frac{(n_o^2 - 1)\sqrt{p} - n_o(p - 1)}{pn_o^2 - 1} \tag{31}$$

To make m_o positive p must be less than n_o^2. This means that stability with respect to temperature can be obtained, because both backward and forward resistance change in the same direction with temperature, and the ratio of temperature coefficients of resistance does not exceed the ratio of backward to forward resistance for most crystal rectifiers.

As an example, take $n_o^2 = 10^4$ which is the approximate value for a crystal diode in the type G9 quad. The relation between the optimum value of m_o and p for this value of n is plotted in Figure 8.

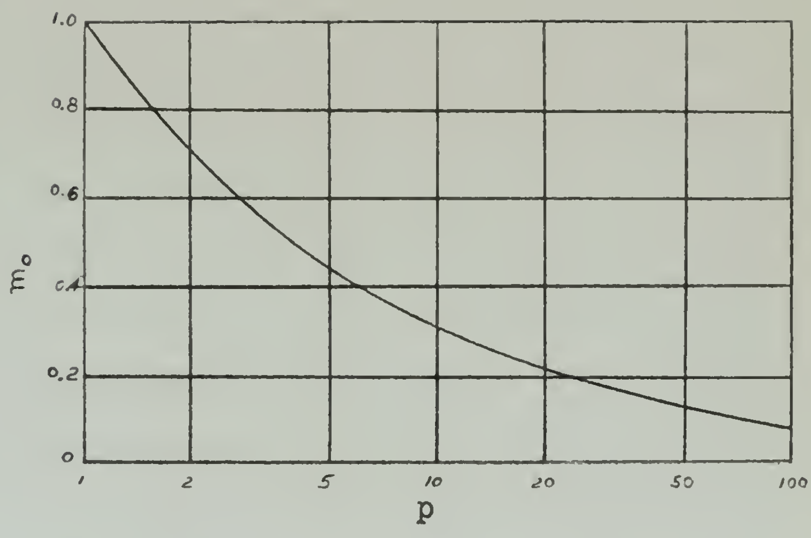


Figure 8. Relation between optimum value of m and p for $n_o^2 = 10^4$.

Choosing a circuit impedance for maximum stability with respect to small variations of restifier resistance decreases the stability with respect to variations of circuit impedance except when p=1. Then the termination for maximum stability is also the termination for maximum efficiency.

If the rectifier elements could be modified to make p=1, the circuit impedance could be chosen on the basis of the modified lattice to provide maximum stability and efficiency coincidently. Tucker* suggests that this be done by shunting each rectifier with a stable resistance such that the variations of effective backward resistance are the same proportionately as the variations of the forward resistance. Then, effectively, $p = 1$, and the design using $m_o = 1$ (based on the modified rectifier resistances) is sumultaneously the most stable in all respects. Also, it is possible for the resistance shunts to compensate largely for the variation of resistance from one rectifier to another. This method appears to be quite satisfactory if the additional loss can be tolorated.

*D. G. Tucker (4)

In the previous chapter R_f and R_b were considered to be constant a.c. resistances presented to an applied signal and switched by the carrier voltage. The modulating function, φ, in this case was independent of time. This was a fair approximation since the circuit parameters are normally adjusted to approach this condition as closely as possible. In practice the a.c. resistance presented to an applied signal is a function of the carrier voltage and therefore of time.

1. The rectifier resistance function

Figure 9 shows a typical current-voltage characteristic for a germanium crystal diode. There is a distinct value of d.c. resistance, $R = \frac{v}{i}$, for each instantaneous value of carrier voltage. Also there is a value of a.c. resistance, $r = \frac{\Delta v}{\Delta i}$, which is not only a function of the carrier voltage but of the signal amplitude as well. This added complication will be avoided here by assuming the signal amplitude is small so that $\frac{dv}{di}$ is actually the a.c. resistance presented to the signal.

Some resistance function, then, is needed to represent the non-linear characteristic of the crystal rectifier. Tucker* suggests an exponential function of the form

$$R = R_o + k\,\epsilon^{-\alpha v}$$

where v is the voltage across the rectifier, R_o is a constant

*D. G. Tucker (3)

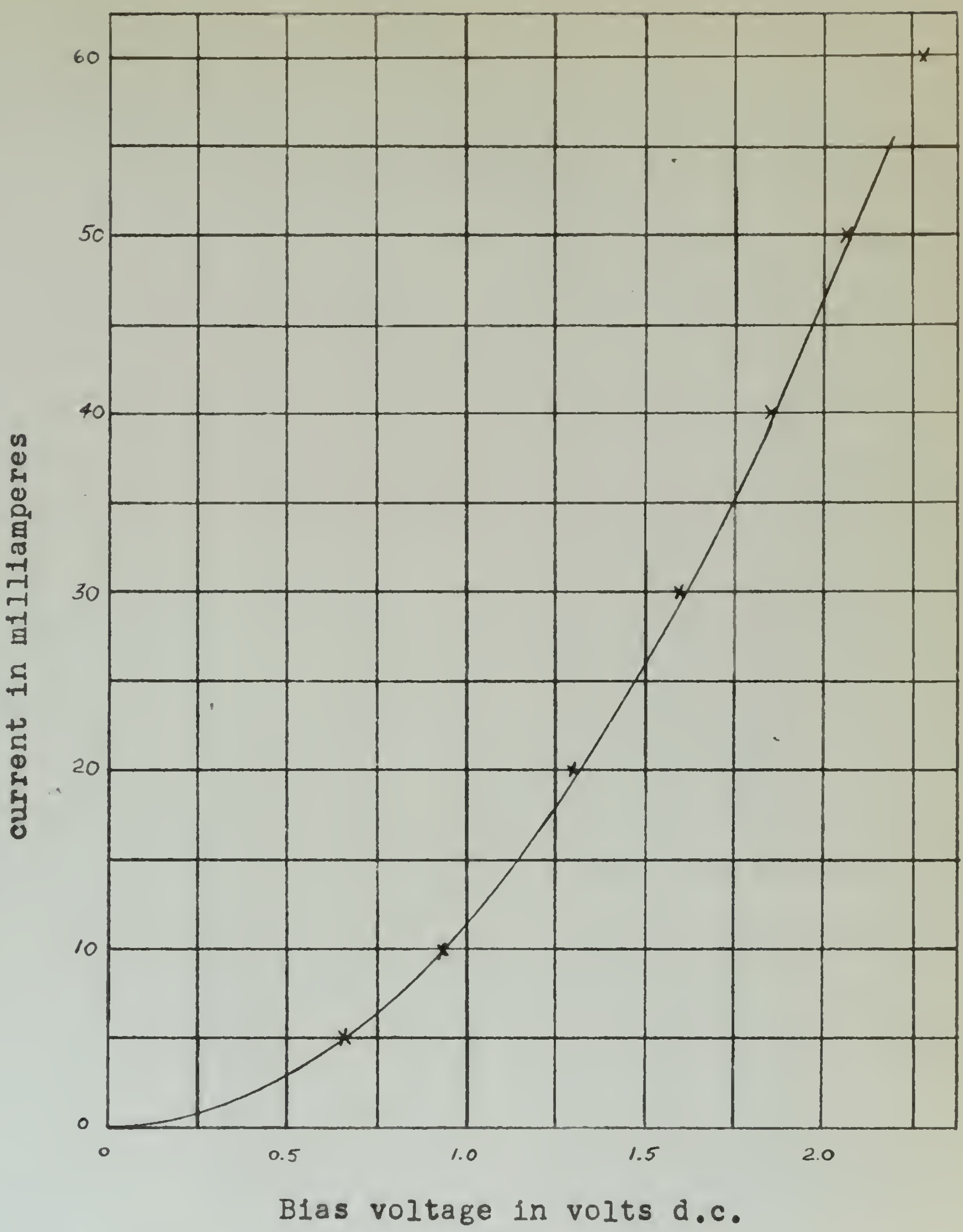


Figure 9. Average current-voltage characteristic of four crystal diodes in type G9 quad.

resistance reached by the rectifier at large values of carrier voltage, k and α are constants determined by the shape of the characteristic. The constants will be different for the backward and forward resistance laws. He goes on to show that this resistance law fits vacuum diodes very well, copper oxide and selenium rectifiers fairly well, and does not fit the crystal diodes very well.

The writer has found that a function of the form $v = ki^n$ fits the current-voltage characteristic of at least some crystal rectifiers very closely. For example, $v = 9.26i^{0.502}$ plots almost exactly on the measured average forward characteristic of the four crystal diodes in a type G9 quad as shown by the x's in Figure 9, while $v = 25.6 \times 10^{14} i^{2.7}$ represents quite closely the average backward characteristic for the same group of crystals.

It might be well to point out that the accuracy of representation of the backward resistance is not as important as that of the forward resistance since the circuit operation will be largely dominated by the two conducting diodes. All that is essentially required is that the backward resistance be high.

Functions of this type will certainly fit an average current-voltage characteristic more closely than the characteristics of several rectifiers taken at random.

Letting $v = ki^n$ be the law to be used, the instantaneous d.c. resistance is given by

$$R = \frac{v}{i} = ki^{(n-1)} \tag{32}$$

and the corresponding a.c. voltage is

$$r = \frac{dv}{di} = nki^{(n-1)} \tag{33}$$

It will be more convenient later to use a resistance law in terms of v, the voltage across the rectifier. The resistance laws given in (32) and (33) may be manipulated to give

$$R_f = \alpha_1 v^{m_1}, \qquad R_b = \alpha_2 v^{m_2}$$

$$r_f = n_1 \alpha_1 v^{m_1}, \qquad r_b = n_2 \alpha_2 v^{m_2}$$

where

$$m = \frac{n-1}{n}, \qquad \alpha = k^{(1-m)}$$

2. The modulating function

Now, the resistance presented to the carrier supply by the lattice will be the parallel combination of the four rectifiers. If $e_c = E_c \sin w_c t$ is the carrier supply voltage and R_c is the impedance of the carrier supply, the ratio v/e_c will be given by

$$\frac{v}{e_c} = \frac{\dfrac{R_b R_f}{2(R_f + R_b)}}{R_c + \dfrac{R_f R_b}{2(R_f + R_b)}} = \frac{R_b R_f}{2R_c(R_b + R_f) + R_b R_f} \tag{34}$$

Substituting the resistance functions for R_b and R_f gives

$$v = \frac{\alpha_1 \alpha_2 v^{(m_1 + m_2)} E_c \sin w_c t}{\alpha_1 \alpha_2 v^{(m_1 + m_2)} + 2R_c\left[\alpha_1 v^{m_1} + \alpha_2 v^{m_2}\right]} \tag{35}$$

This can be solved graphically plotting v against $w_c t$.

The modulating function, repeated here for convenience, now becomes a function of time since R_b and R_f are functions of time.

$$\varphi(t) = \frac{Z(R_b - R_f)}{(Z + R_b)(Z + R_f)} \tag{36}$$

Replacing R_b and R_f by their a.c. resistance functions gives

$$\varphi(t) = \frac{Z(n_2\alpha_2 v^{m_2} - n_1\alpha_1 v^{m_1})}{(Z + n_1\alpha_1 v^{m_1})(Z + n_2\alpha_2 v^{m_2})} \tag{37}$$

Inserting values of v determined from (35) in this equation, $\varphi(t)$ may also be plotted against the angular value of $w_c t$.

Figures 10, 11, and 12 show calculated modulating functions for a ring modulator using the type G9 quad under various conditions of carrier supply and circuit impedance. The shape of the modulating function depends on the following factors:

(a) The rectifier current-voltage characteristic.

(b) The peak amplitude of the carrier voltage (assumed sinusoidal).

(c) The resistance of the circuit supplying the carrier.

(d) The impedance of the circuit in which the modulator is used.

The peak carrier amplitude is usually made large enough so that the rectifiers present a substantially constant forward resistance over the greater part of the appropriate half cycle of the carrier. The modulating function approaches

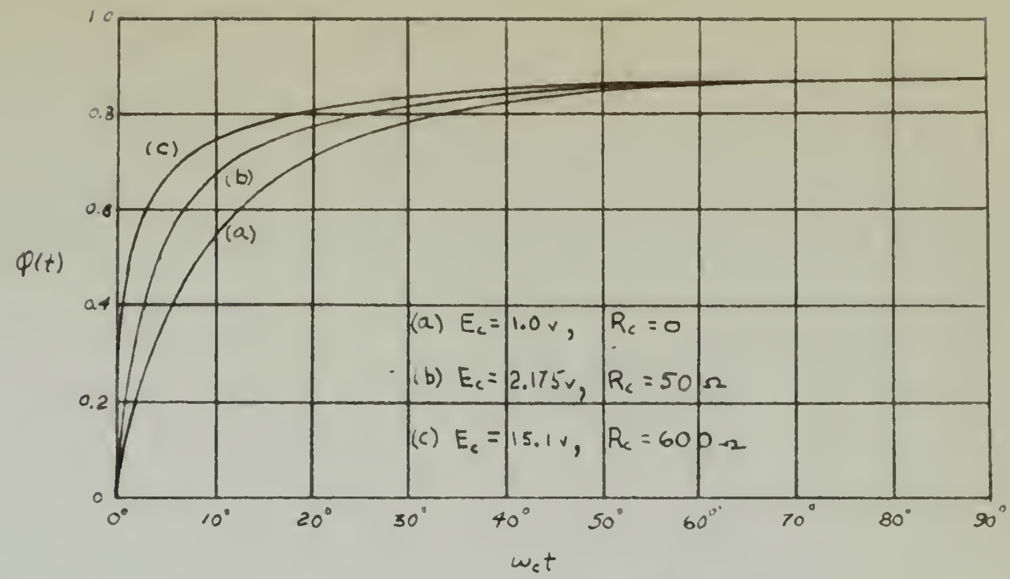


Figure 10. Calculated modulating function for ring modulator using crystal rectifiers having laws $R_f = 85v^{-1}$ and $R_b = 1.4 \times 10^6\, v^{0.6}$.

E_c = Peak value of carrier supply voltage
R_c = Resistance of carrier generator
Circuit impedance = 600 Ω.

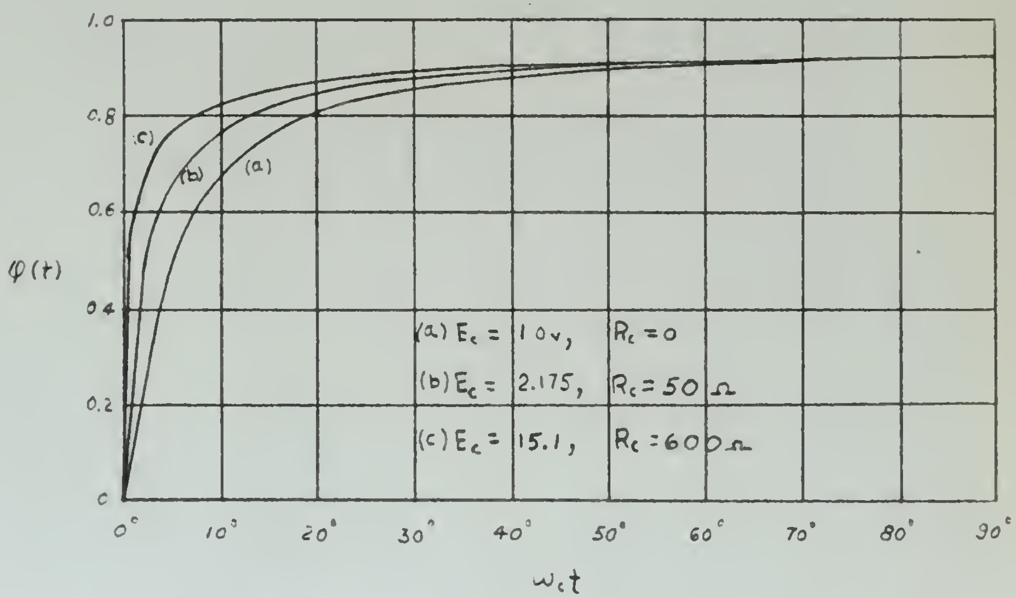


Figure 11. Same as Figure 10 except the circuit impedance is 1000 Ω.

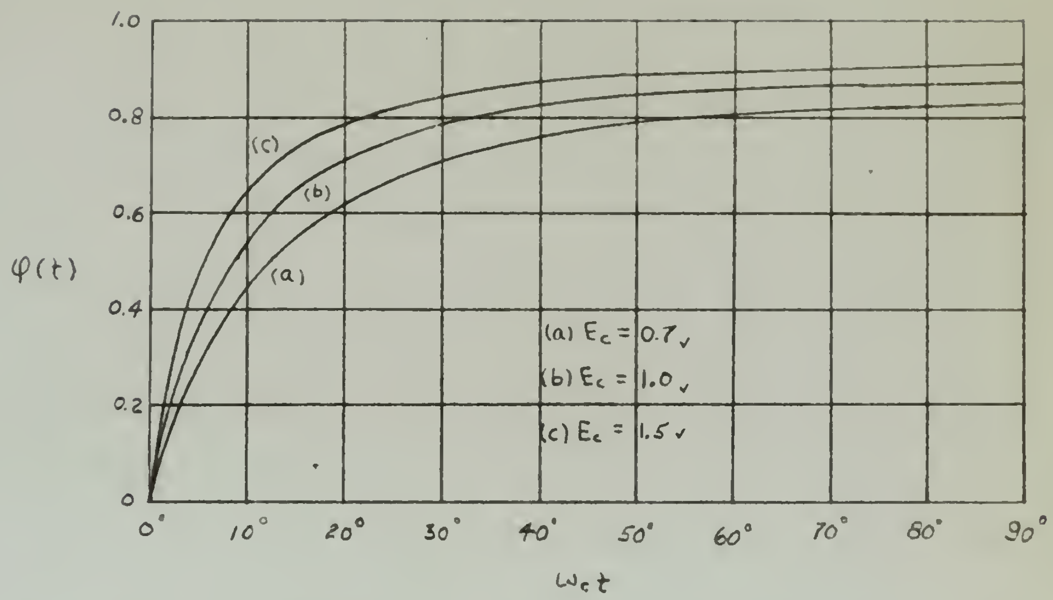


Figure 12. Calculated modulating function with rectifiers of Figure 10 showing effect of varying value of carrier supply voltage (E_c).

Circuit impedance = 600 Ω
Resistance of carrier generator = 0

more nearly a square waveform as either the resistance of the carrier supply circuit or the impedance of the terminating circuit is increased. The use of a more nearly square modulating function is rarely an advantage in itself since the harmonic content of the output will be increased introducing more higher order modulation products which are generally not wanted. However the modulation efficiency is increased under this condition of operation for two reasons. First, The transit time through intermediate values of rectifier resistance is reduced. Secondly, the fundamental component in a square wave is $4/\pi$ times that of a sine wave of the same maximum amplitude. Therefore the efficiency increases as the modulating function approaches a square wave, the peak amplitude remaining constant. For this same reason variations of carrier voltage will have less effect on the efficiency if the carrier supply resistance is high.

This modulation function is not a ficticious mathematical tool. It may be examined by applying a small d.c. signal to the input of the modulator and observing the output on a cathode-ray oscilloscope. In other words $\phi(t)$ is the output of the modulator when a zero-frequency signal is applied to the circuit. If the modulating function is analyzed into a Fourior series by some graphical or empirical method, the relative amplitude of the components of the output signal of the type $nf_c \pm f$ may be determined, where f is the signal frequency.

3. Circuit balance and carrier suppression.

Although the balancing action of the circuit should prevent the signal at any one branch from appearing at either of the other two branches, this ideal performance is not realized in practice. To closely approach this condition puts stringent design requirements on the transformers and requires excellently matched crystals. Crystals are commercially available in matched pairs and in quads consisting of two matched pairs mounted in a metal tube envelope. Examples are the General Electric type G9 quad and the Sylvania 1N71 varistor. These crystals are matched for a specific value of forward current however, and the balance of the crystals varies with temperature and with the current through them. Figure 13 shows experimental results obtained in a test of balance variation with carrier level of the General Electric type G9 quad.

Since the crystal diodes are not matched at every point on their characteristics, a balance obtained for a particular value of carrier voltage will not necessarily be the best balance for other values of carrier voltage. In Figure 13 the circuit was initially balanced with a 1 volt carrier supply and this adjustment maintained during the readings. As may be seen the suppression is even better for lower values of carrier voltage. The writer has found that constant carrier suppression cannot be obtained over a range of carrier voltage even by adjusting the balance for each value of

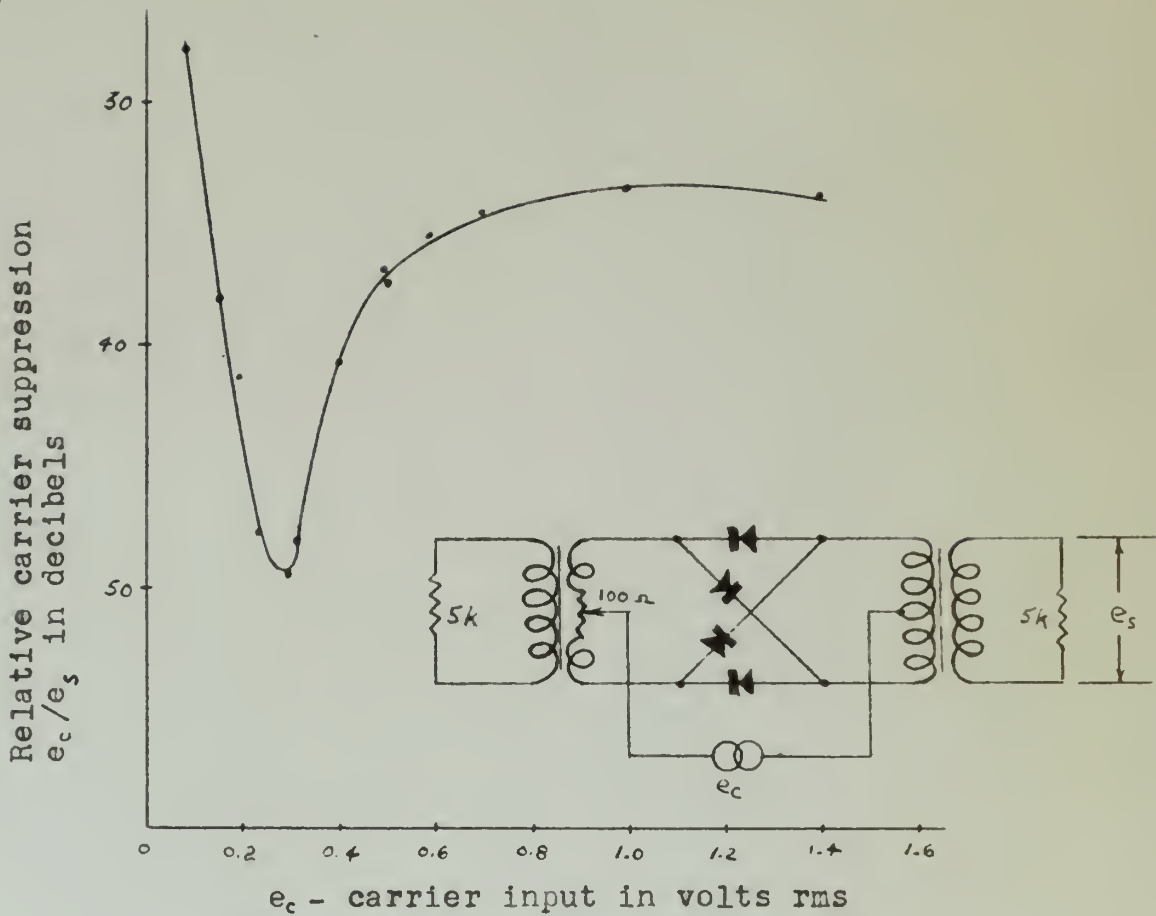


Figure 13. Variation of balance with carrier level.

carrier voltage. The variation of carrier suppression appears to be largely due to variations in departure of the individual rectifier characteristics from the average as the carrier voltage goes through a cycle.

Other factors which are important to the stability of balance are harmonics in the carrier waveform, variations in carrier supply impedance, and slight heating of the rectifiers upon application of the carrier voltage.

At best, balancing with a potentiometer is an averaging process since it is not possible to balance both half-cycles of carrier with the same adjustment, different pairs of rectifiers being involved. Adjusting the potentiometer for minimum fundamental and odd harmonics may increase even harmonic leak. This can be largely overcome by using a balancing potentiometer at each end of the modulator. The writer has found that this type of balance is very unstable and critical of adjustment. The balancing adjustment compensates mainly for the low resistance portion of the rectifier characteristic and the carrier leak occurs mostly during the parts of the carrier cycle when the carrier voltages are low. This then is another reason for using a modulating function which approaches a square waveform. Tucker* gives a more detailed discussion of this effect.

In general, even with extremely well balanced transformers and matched diodes, some auxiliary method of circuit bal-

*D. G. Tucker (4)

ance is necessary to obtain good carrier suppression. Figure 14 shows two possible balancing arrangements. The series arrangement requires a transformer with a split center tap, while the shunt balancing arrangement requires no center tap. From the standpoint of reducing signal losses in the balancing potentiometer, R should be small in the series arrangement and very large in the shunt arrangement. The same requirements

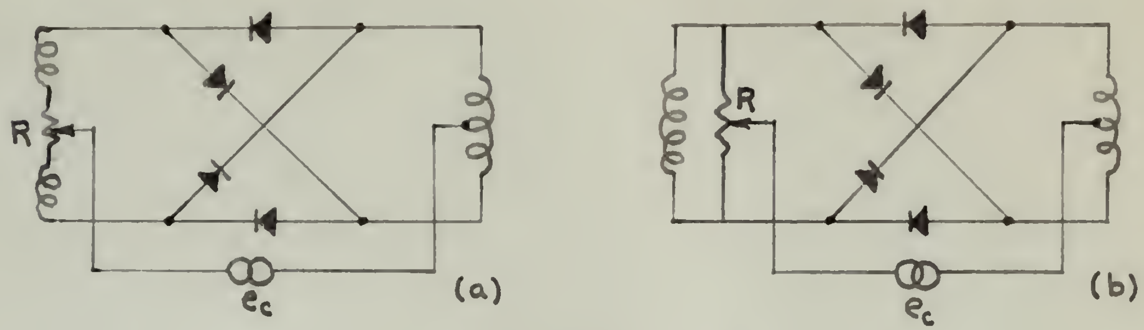
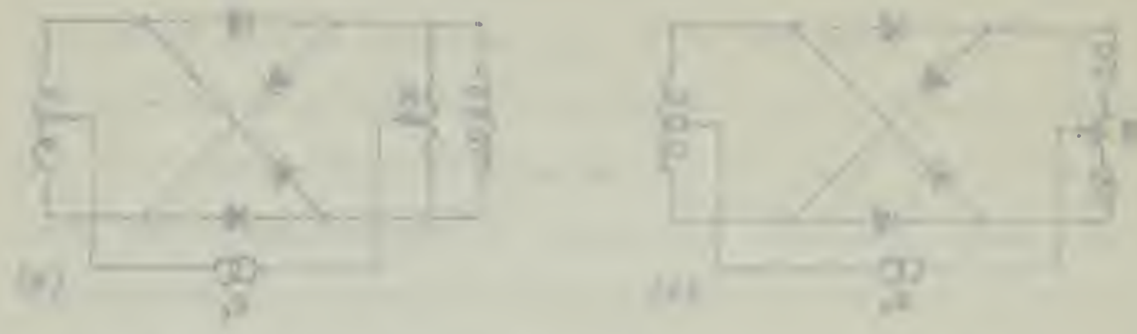


Figure 14. Balancing arrangements, (a) series, (b) shunt.

also hold from the standpoint of not interferring with an established impedance level. In either case R has the effect of increasing the impedance of the carrier source.

One technique for reducing the effect of variations of crystal resistance on circuit balance is to modify the lattice by inserting a resistor in series with each crystal diode. Either of the balancing arrangements achieves essentially the same result since the balancing potentiometers decrease the effect of diode resistance variations on the total resistance in the carrier current path. This compensates largely for variations of the rectifier forward resistance and has practically no effect on the rectifier backward resistance.



Tucker gives what appears to be an excellent method of selection of rectifiers for low carrier leak in a ring modulator, and gives results obtained with five rectifiers using this method to select the rectifier elements. The results given indicate that the method is very effective.

V. REFLECTION

The impedances of the connected circuits react on each other similar to the way the terminating impedances of any four-terminal linear network react on each other. These circuits will generally present an approximately constant resistance to the desired signal frequencies but the terminations will be reactive and of varying magnitude to frequencies outside the desired signal band.

Referring to Figure 5 and applying generalized reflection theory* to this four terminal network

$$\frac{e}{I_a} = \frac{Z(\epsilon^{-\gamma} + r_R \epsilon^{\gamma})}{\epsilon^{-\gamma} - r_R \epsilon^{\gamma}} \tag{38}$$

where

$$r_R = \frac{Z_2 - Z}{Z_2 + Z}, \tag{39}$$

$$Z = \sqrt{R_b R_f}, \tag{40}$$

and γ is the propogation constant. The signal impedance is a combination of the characteristic impedance of the lattice and the impedance of the connected circuits at all the modulation frequencies. It can readily be seen that it $Z_1 = Z_2 = Z$ there will be no reflections. The corresponding impedance at the other end of the network may be similarly obtained.

*E. A. Guillemin (9)

The solution for current flow at any frequency can be written exactly like that of the case of linear networks in which the current is expressed as that flowing in a matched circuit modified by a reflection factor. Consider the input signal current I_0 which will be translated to the output current I_{1+} at the upper sideband frequency. From (18) the output current under matched conditions is

$$I_{1+} = \frac{2\varphi}{\pi} I_0 \tag{41}$$

and the total current at the output terminals at the side-band frequency (1+) with reflection will be

$$I_{1+} = \frac{2\varphi}{\pi} I_0 \left[1 - \frac{Z_{1+} - Z}{Z_{1+} + Z}\right] \tag{42}$$

Reflection from any modulation product can be similarly treated.

If these reflections noticeably affect the lattice impedance at the signal frequencies or the lower loss modulation product frequencies, resistance pad separation between the modulator and the external circuits is usually the simplest corrective measure if the increased loss can be tolerated.

VI. CONCLUSION

It has been shown how a proper resistance law to represent the rectifier elements makes it possible to investigate the finer points of modulator performance and to improve the design of the modulator. The use of a modulating function, which is the reciprical of the insertion loss of the modulator, has been shown to be a useful tool in this investigation. The effect of the various circuit parameters on the efficiency and stability of the modulator has been presented, indicating the following general conclusion:

(a) The circuit impedance should be a compromise between a high value for greater efficiency and a low value for less distortion.

(b) Efficiency and stability with respect to variations of carrier supply voltage are increased as the carrier supply resistance is increased, but this also increases the distortion content of the output signal.

(c) The only way to achieve low carrier leak and stability of carrier leak is by a proper method of rectifier selection which matches the entire characteristics of the rectifiers as closely as possible.

It appears that the best all around modulator design would be one in which a good method of rectifier selection is used, such as that proposed by Tucker, and in which stable shunt resistors are used around each rectifier to make the terminating circuit impedance be the value for both maximum efficiency and maximum stability. The result-

ing modulator would have low carrier leak and maximum stability in all respects. Of course some efficiency is lost in accomplishing this design.

The design of a modulator using the techniques presented here is tedious and time consuming. The writer can only reach the conclusion that an empirical design based on the knowledge of the effects of the various circuit parameters is often the most suitable approach to the problem, the more detailed procedure being reserved for circuits requiring laboratory precision.

Usually the quantities of greatest interest in the performance of a balanced modulator are the distortion present in the output signal and the effective carrier suppression. The insertion loss is of interest but of secondary importance.

The effective carrier suppression is the ratio of the desired output signal voltage to the carrier frequency voltage present in the output. Whereas the ratio of input carrier voltage to output carrier voltage may indicate considerable carrier suppression, the effective carrier suppression is a function of the output signal voltage also. As usual, there must be a compromise, since increasing the signal voltage increases the effective carrier suppression but at the same time increases the distortion.

Too large a signal amplitude not only results in the production of undesired frequencies but also causes the impedance and loss characteristics of the modulator to vary with the signal amplitude. In conflict with this require-

ment the effective carrier suppression increases with signal level.

All in all, the modulator design is a series of compromises as in any engineering problem. However, this paper has shown an analytical approach which may lead to improvement in design over the strictly empirical method.

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